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GANIT

GANIT: Journal of Bangladesh Mathematical Society

**GANIT** *J. Bangladesh Math. Soc.* 40.2 (2020) 86–94
DOI: https://doi.org/10.3329/ganit.v40i2.51310

# Study on Flow Behavior of Parallel Lumped-Model under Constant Flow for Bronchial Tree

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## ABSTRACT

Redistribution of flow in the bronchial tree is an important factor that enhances gas exchange in the lungs, especially, in diseased lungs. The bifurcated bronchial tree is like an electric network in series and parallel. A lumped-model of parallel system for constant flow rate is solved analytically to demonstrate the intrinsic characteristics and the dynamic behavior of the system. Inertial and capacitive time constants are calculated for 19$^{th}$ generation airways of human lung to control the solution. The investigation revealed that (i) higher inertial force takes more time to maximize the inertial flow to steady state and more time to minimize the resistive flow to steady state and (ii) the compliant effect is negligible for a relatively higher inertial time constant, $t_L > 6s$ on the same experimental conditions.



**Received:** June 26, 2020 **Accepted:** November 23, 2020 **Published Online:** January 15, 2021

**Keywords:** Parallel lumped-model; Bronchial tree; Inertial flow; Inertial time constant

***AMS Subject Classifications 2020:*** 76Z05; 76Z10; 74A10

## 1. Introduction

Inspiration of $O_2$ (Oxygen) and expiration of $CO_2$ (Carbon dioxide) along human lung is a fluid transportation from trachea (0$^{th}$ generation, G0) to alveolar end (23$^{rd}$ generation, G23) and vice versa. Pressure in steady or pulsatile is a driving force of oscillatory flow of respiration. The relation between pressure and flow in a respiratory tube depends on its diameter, length, roughness and elasticity. Angle and curvature of bifurcations are other factors which influence the pressure-flow relation in bronchial tube. It is very difficult to simulate a problem showing the effect of all parameters in a single experiment. Mathematical modeling, computational model, circuit analogy and lab experiment are the leading methods of approximating the physical values. So, the flow through complex geometry mounts the importance of lumped model with three elements: resistance, inertance and compliance associated with the flow and geometry of lung tube. Numerous works have been done on respiratory flow experimentally and numerically. Otis et al. [1] carried out a study of breathing movements which overcome several types of resisting forces such as viscous and turbulent resistance, resistance for elasticity and deformation of airways. They experimentally measured the resistance related to moving gas through respiratory tract and derived equations that give an approximate description of the mechanical system. They measured resistance related to moving gas through the respiratory tract experimentally and derived equations that give an approximate description of mechanical system. A non-linear lumped parameter model has been developed to investigate the dependency of airflow distribution in the bronchial tree on time

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dependent parameters, resistance and compliance by Elad et al. [2]. They revealed that diameter asymmetry is more dominant than length asymmetry for compliance effect. Moreover, the parameters are defined by Zamir [3]. Saing et al. [4] developed a non-linear multi-compartment lung mechanics model that accounts for non-linearities of the airway resistances and the lung compliances. They also found that the non-linearrity in the airway resistance is much evident during expiration than inspiration. Eckmann et al. [5] and Fujioka et al. [6] studied oscillatory flow of gas transport for curved tubes. They found effective diffusion in bent and bifurcated tubes than in straight tubes and gas exchange between inspiration and expiration. Calay [7] worked for numerical simulation of breathing in normal and exercise conditions. He found that, the respiratory flows are strongly dependent on the convective effect and the viscous effect with some contribution of the unsteadiness effect. Hirahara et al. [8] simulated oscillatory gas flow in lower airways where boundary conditions are based on compliance and resistance of airways. They investigated inertial effect and irreversible flow for various compliances. Also, phase delay was calculated by lumped parameter analysis. The effect of compliance under constant resistance and inertance for lumped parameter model is investigated by Ahmmed [9] . He introduced inertial and capacitive time constants to characterize the oscillatory flow behavior. Various mechanisms of gas transport for oscillatory flow along human lung are identified by Chang [10] and Pillow [11]. They examined the importance of convective transport of gases for gas exchange during respiration. These studies were limited to mathematical modeling and numerical simulation of different transport problems along bronchial tree of human lung. Our efforts has been directed towards the parallel airways of bronchial tree.

In the present work we solved a lumped-parameter model of parallel system to study the dynamics of resistive flow, capacitive flow and inertial flow for RLC elements. The elemental values of resistance, inertance and compliance were evaluated from laboratory studied of airway models and physiological measurements [2]. The change of basic intrinsic characteristics of RLC system due to the change of RLC elements for respiratory disease or clinical intervention, will demonstrate the dynamic behavior of lung problem.

**Nomenclature:**

| Symbol | Meaning | Unit |
|---|---|---|
| $R$ | Resistance | $Pa.s.m^{-3}$ |
| $L$ | Inertance | $Pa.s^{2}.m^{-3}$ |
| $C$ | Compliance | $m^{3}Pa^{-1}$ |
| $q$ | Total flow rate | $m^{3}s^{-1}$ |
| $d$ | Diameter of channel | m |
| $q_R$ | Resistive flow rate | $m^{3}s^{-1}$ |
| $q_L$ | Inertial flow rate | $m^{3}s^{-1}$ |
| $q_C$ | Capacitive flow rate | $m^{3}s^{-1}$ |
| $t_L$ | Inertial time constant | sec (s) |
| $t_C$ | Capacitive time constant | sec (s) |
| $R_L$ | Laminar resistance | $Pa.s.m^{-3}$ |
| $\Delta p$ | Pressure drop | Pa |
| $\Delta v$ | Change in volume | $m^{3}$ |
| $\Delta t$ | Change in time | sec (s) |
| Re | Reynolds number | Dimensionless |

**Greek Letters**

| | | |
|---|---|---|
| $\nu$ | Kinematic viscosity of air | $m^{2}.s^{-1}$ |
| $\mu$ | Dynamic viscosity of air | Pa.s |
| $\rho$ | Density of air | $kg.m^{-3}$ |
| $\beta$ | Roots of solution | $s^{-1}$ |

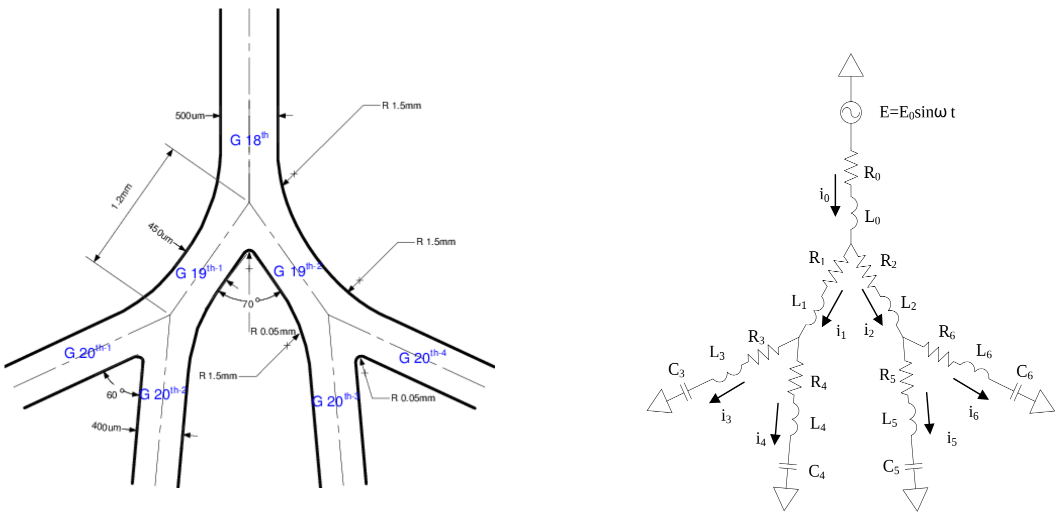
## 2. Anatomy and Model Channel

### *2.1 Anatomy of Human Lung*

According to dichotomy model of human lung, the bronchial tree consists of 23 generations from trachea ($0^{th}$ generation, G0) to alveolar end ($23^{rd}$ generation, G23), G for generation and there are mainly two states; gas flow state and gas diffusion state in the human lung Weibel [12]. Gas mainly transports from trachea (G0) to the terminal bronchioles (G16) which is called the conducting zone or anatomical dead space. The zone from G17 to G23 is called respiratory zone where convection and diffusion take place in gas transportation. The transition from convection to diffusion occurs at the beginning of respiratory zone and gas exchange mostly occurs at the end of respiratory zone in presence of respiratory bronchioles, alveolar ducts and alveolar sacs. We expect active gas mixing and gas exchange in the respiratory zone so that we prefer the generations G18 to G20 for numerical experiment.

### *2.2 Model Channel of Human Lung Structure*

The bronchial airways of human lung are compliant by nature. Because of experimental limitations, it is quite impossible to conduct experiment in lab with a real lung. For this, we propose a rigid model channel connecting compliant balloon as alveoli at the end of 20$^{th}$ generation. The lab experimental value of resistance, inertance and compliance are incorporated in numerical experiment to have the real flow characteristics of our model channel. Also, the geometric configuration and dynamical behavior is considered in the entire experiment. Fig. 1.1(a) illustrates doubly-bifurcated two-dimensional respiratory model channel of G18 to G20 for adult human lung and Fig. 1.1(b) is the corresponding circuit analogy of lumped-model. The primary feature of our model channel is symmetric bifurcation of one tube into two smaller tubes. Each branch of channel was fabricated axially symmetric and entire channel is symmetric about axis of mother tube (G18). The ratio of diameters of our test channel, $d_{18} : d_{19} : d_{20} \equiv 10:9:8$ shows the linear change of flow channel. The widths (rectangular channel) as well as the diameters (cylindrical channel) of G18, G19 and G20 are 500, 450 and 400μm, respectively. The average width of rectangular channel is 500μm that represents the depth of our channel. The length of the middle branch, G19 is1.2 mm along the centreline and the lengths of G18 and G20 are enough to reach up to fully developed flow. Since channel resistance is delicately sensitive to change in radius, we take into consideration the real scale measurement of G19. So, real flow phenomena and hence gas exchange mechanisms are expected in G19.




(a) Physical model channel (b) RLC circuit model

**Figure1.1** Model Channel of 18$^{th}$ to 20$^{th}$ generation

## 3. Lumped-Parameter Model

### *3.1. Description of Model Equations*

The pressure-flow relation in a bifurcated model channel of human lung depends on channel diameter, length, angle of bifurcation, the curvature at junction, the elasticity of channel and the driving pressure (steady or oscillatory) applied along it. The flow through complex geometry mounts the importance of lumped model. The flow simulation of our lumped model consists of three parameters such as resistance (*R*), inertance (*L*) and compliance (*C*). Resistance is a measure of pressure difference in a fluid acting opposite to the motion of fluid in a channel. It is proportional to the pressure drop ( $\Delta p$ ) as driving force in a channel for a resistive flow rate ( $q_R$ ). The resistance of channel may be defined as

$$\Delta p = Rq_R \tag{3.1}$$

Inertance (*L*) is a measure of pressure difference in a fluid required to change the inductive flow rate ( $q_L$ ) with time. Inertia in flow field occurs due to the acceleration or deceleration. It can be defined by

$$\Delta p = L\frac{dq_L}{dt} \tag{3.2}$$

Compliance (*C*) of a bronchial channel of human lung is the ability to stretch and expand. A balloon like an alveolus of lung is in an inflated state when pressure inside the balloon is higher than pressure outside it. The compliance of the balloon is defined by the amount of change in the pressure difference ( $\Delta p$) required to produce a change in volume ( $\Delta v$ ) such that

$$C = \frac{\Delta v}{\Delta(\Delta p)} \tag{3.3}$$

For incompressible flow, the only way to change the volume of the balloon is to change the amount of fluid within it. If a small change in volume ( $\Delta v$ ) for an increment time ( $\Delta t$ ) is found for a volumetric or capacitive flow rate ( $q_C$ ), the relation becomes

$$\Delta v = q_C \Delta t \tag{3.4}$$

Equations (3.3) and (3.4) lead to

$$\Delta(\Delta p) = \frac{1}{C} q_C \Delta t \tag{3.5}$$

Due to oscillatory flow and complex geometry of bronchial channel, the driving pressure varies with time. As a consequence, if $\Delta p = \Delta p(t)$ then $q_C = q_C(t)$ and Eqn. (3.5) becomes

$$\Delta p = \frac{1}{C}\int q_C \, dt \tag{3.6}$$

Now we consider that the lumped parameters *R*, *L* and *C* are in parallel under a driving pressure drop ( $\Delta p$) as shown in Fig.1.1 where G19L and G19R are parallel in 19th generation and G20LL, G20LR, G20RL and G20RR are parallel in 20th generation. Total flow rate into the parallel system is given by

$$\begin{aligned} q &= q_R + q_L + q_C \\ &= \frac{\Delta p}{R} + \frac{1}{L}\int \Delta p \, dt + C\frac{d(\Delta p)}{dt} \end{aligned} \tag{3.7}$$

Differentiating once with respect to time and introducing inertial time constant, $t_L (= L/R)$ as well as compliant time constant, $t_C (= RC)$ we yield

$$t_C \frac{d^2(\Delta p)}{dt^2} + \frac{d(\Delta p)}{dt} + \frac{1}{t_L}\Delta p = R\frac{dq}{dt} \tag{3.8}$$

### *3.2. Parameters and RLC Elements*

The Reynolds number (Re), an important dimensionless quantity, is a ratio of inertial forces to viscous forces within a fluid which is used to predict flow patterns in different fluid flow situations. The Reynolds number is defined as

$$\mathrm{Re} = \frac{4q}{\nu \pi d} \tag{3.9}$$

where $q$ , $\nu$ and $d$ are total flow rate, kinematic viscosity of air and diameter of the tube. The resistance at a given generation for the total flow rate is defined as [13]

$$\begin{aligned} R &= R_L\left(0.556 + 0.067\,\mathrm{Re}^{1/2}\right) \\ R_L &= \frac{128\mu l}{2^n \pi d^4} \end{aligned} \tag{3.10}$$

Here, $R_L$ is the laminar resistance in *n*-th branch tube, *l* is the length of the tube and $\mu$ *is* the viscosity of air. The inertance is an associated parameter which corresponds to the fluid acceleration or deceleration and can be expressed in terms of tube dimensions[14], thus

$$L = \frac{4\rho l}{\pi d^2} \tag{3.11}$$

where, $\rho$ is the density of air. Our interest of experiment is in G18 to G20. The bronchial airways of G20 are connected with compliant tubes as volume-dependent compliance, whereas G18 and G19 preserve constant volume for rigidity. According to the investigation of Sharp et al.[15], the net lung's compliance (C) is :

$$C = \frac{\Delta v}{\Delta(\Delta P)} = 0.128 \sim 0.31\left[\mathrm{L/cmH_2O}\right] \tag{3.12}$$

The compliance below G20 was estimated from the number of branches and the length of each tube that was adjusted for the equivalent compliance.

### 3.3. Solution of Governing Equation

If total flow rate is kept constant ( $q = \text{const.}$ ) to find the viscous effect of resistive tube, the inertial effect of inductive tube and the capacitive effect of compliant tube, then Eqn. (3.8) grow into

$$t_C \frac{d^2(\Delta p)}{dt^2} + \frac{d(\Delta p)}{dt} + \frac{1}{t_L}\Delta p = 0 \tag{3.13}$$

Eqn. (13) is a standard second order differential equation with constant coefficients. Its solution depends on the nature of the roots of the associated equation

$$t_C \beta^2 + \beta + \frac{1}{t_L} = 0 \tag{3.14}$$

The roots are generally given by

$$\beta = \frac{-1 \pm \sqrt{1-(4t_C/t_L)}}{2t_C}$$

In case of over damped flow condition, $\beta$ is real for $4t_C < t_L$. If $\beta_1$ and $\beta_2$ are two distinct real roots, then solution of Eqn. (13) is

$$\Delta p(t) = Ae^{\beta_1 t} + Be^{\beta_2 t} \tag{3.15}$$

Where A and B are arbitrary constants to be calculated. The non-dimensional forms of resistive, inductive and capacitive flow rates are $\bar{q}_{R,L,C}(t) = \dfrac{q_{R,L,C}(t)}{q}$ such that

$$\begin{aligned}
\bar{q}_R(t) &= \bar{A}e^{\beta_1 t} + \bar{B}e^{\beta_2 t} \\
\bar{q}_L(t) &= (\bar{A}e^{\beta_1 t}/\beta_1 + \bar{B}e^{\beta_2 t}/\beta_2 + t_L)/t_L \\
\bar{q}_C(t) &= (\bar{A}\beta_1 e^{\beta_1 t} + \bar{B}\beta_2 e^{\beta_2 t})t_C
\end{aligned} \qquad \text{3.16 (a, b, c))}$$

The pressure drop can also be put in non- dimensional form as

$$\overline{\Delta p}(t) = \Delta p / Rq = \bar{A}e^{\beta_1 t} + \bar{B}e^{\beta_2 t} \tag{3.17}$$

Where, $\bar{A} = A/Rq$ and $\bar{B} = B/Rq$

At initial condition, $t = 0$ Eqns. (16) reduce to

$$\bar{q}_R(0) = \bar{A} + \bar{B}$$

3.18 (a, b, c)

$$\bar{q}_L(0) = (\bar{A}/\beta_1 + \bar{B}/\beta_2 + t_L)/t_L$$

$$\bar{q}_C(0) = (\bar{A}\beta_1 + \bar{B}\beta_2)t_C$$

We need only two initial conditions to calculate two unknown constants A and B. So, we may prescribe only two initial flow rates. We intend to examine the dynamic system of our problem for $t_L$ and $t_C$ so that the entire inflow *q* is passed through the resistive tube while the sum of the inductive and capacitive flow rates is zero, that is

$$\bar{q}_R(0) = 1,\ \bar{q}_L(0) + \bar{q}_C(0) = 0 \tag{3.19}$$

The sum of two initial flow rates is zero only if they are separately zero and hence the initial conditions are

$$\bar{q}_R(0) = 1,\ \bar{q}_L(0) = 0,\ \bar{q}_C(0) = 0 \tag{3.20}$$

Which give, after some algebra

$$\bar{A} = \beta_2/(\beta_2 - \beta_1),\ \bar{B} = -\beta_1/(\beta_2 - \beta_1) \tag{3.21}$$

The values in (3.21) satisfy the conditions $\bar{A} + \bar{B} = 1$ in 14(a) and $\bar{q}(0) = \bar{q}_R(0) + \bar{q}_L(0) + \bar{q}_C(0) = 1$in (3.7). With the help of values of $\bar{A}$ and $\bar{B}$ , the non-dimensional flow rates in Eqns. (3.18) can be plotted as functions of time. Values of the time constants $t_L$ , $t_C$ are required in order to complete the process.

## 4. Results and Discussion

Electrical analogy of parallel flow for over-damped condition is considered in our present study by which we examine the flow behavior along human lung. It is very difficult even impossible to attain quantitative results instead of qualitative behavior. Total flow is kept constant for model equation (3.8) with initial conditions (3.20). Resistive flow rate ( $q_R$ ) through the resistive tube, inertial flow rate ( $q_L$ ) through the inductive tube and capacitive flow rate ( $q_C$ ) through the capacitive tube are taken in over-damped flow. The intensity of flow through inductive tube and capacitive tube is measured by inertial time constant ( $t_L$ ) and capacitive time constant ( $t_C$ ), respectively.

The time constants are calculated from the values $L = 8.75\times10^7$[Pa.s$^2$/m$^3$], $R = 1.44\times10^7$[Pa.s/m$^3$] and $C = 6.94\times10^{-8}$[m$^3$/Pa] obtained from Eqns. 10-12. The values of $t_L$ , $t_C$ are required in order to complete the process graphically. The total flow into the system is fixed at a normalized value of 1 and the initial values of $q_L$ , $q_C$ and $q_R$ are set to 0, 0 and 1 respectively. So, initially the resistive tube carries the entire flow rate into the system. Then the numerical experiment is continued for $t_L = 6$ sec and $t_C = 0.99$ sec within $4\,t_C < t_L$.The non-dimensional flow rates in Eqns. 16 are plotted as functions of time as shown in Fig. 4.1. Here the inertial flow, $q_L$ (dashed dot curve, green) grows rapidly from zero to the entire flow into the system and $q_C$ , $q_R$ diminish to zero as a consequence. The inertial flow rate approaches to steady state at $q = 1$ for $t_L$ =6 s and capacitive flow rate (dotted curve, blue) diminishes to zero at $q = 0$ for $t_C = 0.99$ s. Moreover, the resistive flow rate (solid curve, black) becomes zero in the system.

The effect of capacitance like compliance (*C*) of the system is evident in capacitive flow rate curve for a minimum time constant, $t_L$ =6 s and $t_C = 0.99$ s as in Fig. 4.1. The curve shows that, the capacitive flow rate is initially zero and ultimate to zero but capacitor is inflated prior to be steady state at zero. That means the capacitor absorbs fluid mass prior to its ultimate value to zero due to a very short time constant of $t_L$ =6 s. The larger value of $t_L\,(10L)$ is considered in Fig.4.2 to find the state of capacitor in the system. In time constants $t_L(= L/R)$ and $t_C(= RC)$ , *R* for tube resistance and *C* for capacitance of a system are taken constant as experimental value. So, $t_C = 0.99$s is constant throughout the experiment and $t_L$ varies for the value of inertance (*L*). Mathematically, $t_L = \frac{1}{R} f(iL), 1 \le i \le 10$ gives the pairs $(iL/R, t_L) = (1, 6), (2, 12), ..., (10, 60)$. For $(10L/R, t_L)$ and $t_C = 0.99$s , the effect of capacitance is negligible due to higher inertial effect as depicted in Fig. 4.2. The higher the inertial effect the longer it takes the flow to reach steady state.

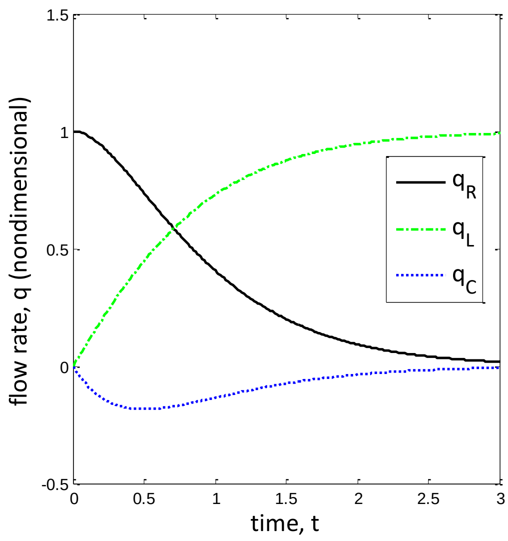



**Fig. 4.1** Flow behavior of $q_R$ , $q_L$ and $q_C$ for $t_L = 6,\ t_C = 0.99$ s.

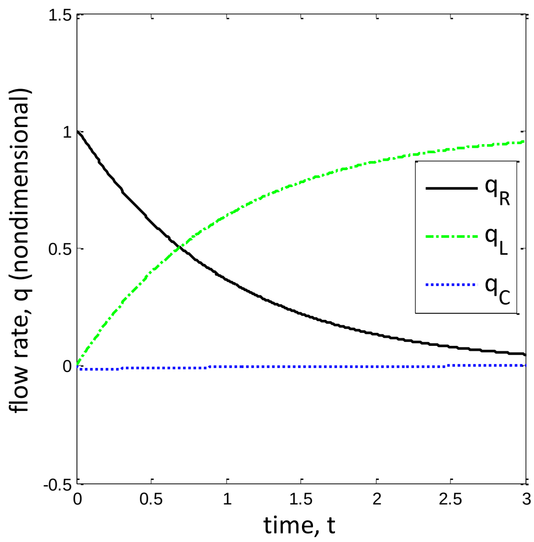



**Fig. 4.2** Flow behavior of $q_R$ , $q_L$ and $q_C$ for $t_L = 60,\ t_C = 0.99$ s.

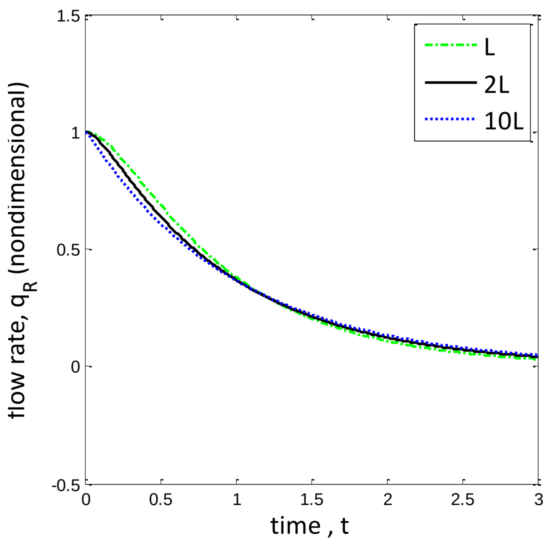



**Fig. 4.3** Variation of resistive flow, $q_R$ with the change of $t_L$ .

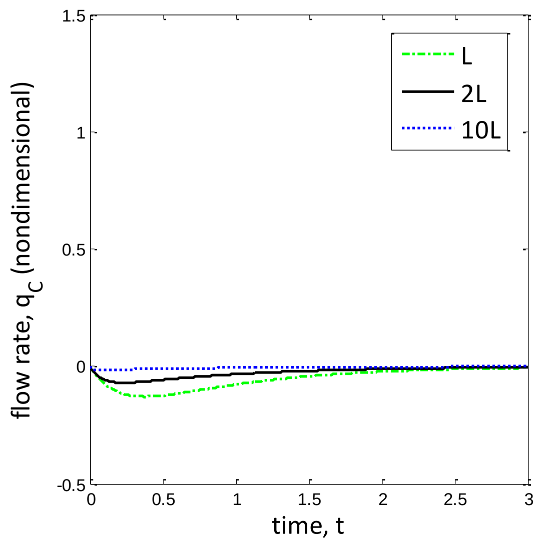



**Fig. 4.4** Variation of capacitive flow, $q_C$ with the change of $t_L$ .

From Fig. 4.1 and 4.2 we can conclude that shorter the inertial time constant, higher the effect of capacitance in parallel network of lung model. To investigate the effect of inertia is a challenging work in biological field especially within 18$^{th}$ to 20th generation of human lung due to high resistance and compliance properties. The rate of gas exchange within lung depends on the rate of inertia and diffusion. It is also required to examine the effect of inertia individually in resistive tube, in capacitive tube and in inductive tube when the total flow is constant in the parallel system.

Fig.4.3 shows the variation of $q_R$ for different values of $L$ when $t_L = f(iL),\ i \in (0, \infty)$ under constant flow. We see that the gradient of resistive flow is gradually decreasing with the increase of inertial force. It means higher inertial force takes more time to minimize the resistive flow rate at $q(t) = 0$ of steady state.

Fig.4.4 shows the variation of $q_C$ for different values of $L$ under constant flow rate. Mathematically, this effect is found from Eqn. (16c). The gradient of flow curve is initially negative and then positive to reach steady state at zero. be steady state to zero. The variations of $q_C$ indicates that compliant tube receives fluid particles and discharge it within a very small inertial time constant (L/R). In a consequence, the compliant effect is negligible for a long inertial time constant as in Fig. 4.4 (blue curve).

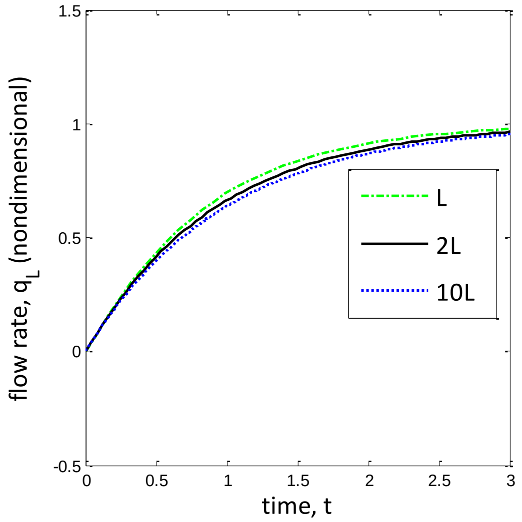



**Fig. 4.5** Variation of inertial flow, $q_L$ with the change of $t_L$.

Fig.4.5 shows the variation of $q_L$ for different values of $L$ when $t_L = f(iL),\, i \in (0, \infty)$ under constant flow rate. We see that the gradient of inertial flow is gradually decreasing with the increase of inertial force. It means higher inertial force takes more time to maximize the inertial flow rate at $q(t) = 1$ of steady state. In a word, the resistive flow in Fig.4.3 and the inertial flow in Fig.4.5 behave reversely as shown in Fig. 4.1 and 4.2.

## 5. Conclusions

A lumped-model of parallel system for incompressible air of constant inflow through bronchial tree is considered. The movement of air is associated with inertance (*L*), the viscosity of air is subject to resistance (*R*) of tube wall and the elasticity of tube is associated with compliance or capacitance (*C*) of bronchial tube. Free dynamics of the RLC system represent its basic intrinsic characteristics. Any relevant change of *R*, *L*, *C*, as a result of respiratory disease or clinical intervention, may change the intrinsic characteristics and demonstrate the dynamic behavior of the system. We found that the inertial flow ultimately encompasses the entire flow into the system, thus reducing resistive and capacitive flow to zero. The dynamics of resistive flow, capacitive flow and inertial flow are examined for the variant inertial forces. We found that higher inertial force takes more time to maximize the inertial flow to steady state ( $q(t) = 1$) and more timeto minimize the resistive flowto steady state ( $q(t) = 0$ ). It is also found that the compliant effect is negligible for a relatively high inertial time constant, $t_L = 60s$ on the same experimental conditions.

## Acknowledgements

The authors are grateful to the anonymous referees for their valuable comments and constructive suggestions of the manuscript.

## Funding

The authors declare that there was no fund available.

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